

Butterworth Filters

Polynomials, Poles and Circuit Elements

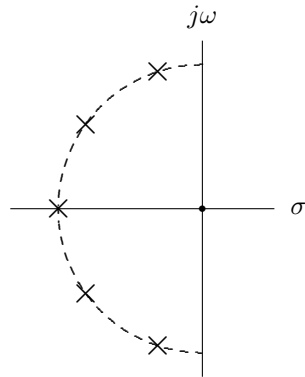
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This document contains tables of reference values for filter designers involved in the design of Butterworth filters. The tables are new and represent the state of the art in machine calculation of filter parameters.

Polynomials, poles and element values are presented for normalized Butterworth filters having a 3dB cutoff frequency of 1 radian per second. In order to transform these values for a particular application it will be necessary to perform frequency and/or impedance scaling. Simple methods and formulas for such scaling can be found in any filter design text.

Element values are provided for implementing LC ladder solutions for each filter. The particular values given are for filters with equal source and load terminations, where R_S and R_L equal 1Ω . Because of the large number of possible source and termination resistances, the case of unequal terminations is better handled by a computer program than by tables.

Butterworth Pole Locations



Pole locations for
5th order
Butterworth filter

Order	σ	$j\omega$
1	-1.0000000000	0.0000000000
2	-0.7071067812	± 0.7071067812
3	-0.5000000000 -1.0000000000	± 0.8660254038 0.0000000000
4	-0.3826834324 -0.9238795325	± 0.9238795325 ± 0.3826834324
5	-0.3090169944 -0.8090169944 -1.0000000000	± 0.9510565163 ± 0.5877852523 0.0000000000
6	-0.2588190451 -0.7071067812 -0.9659258263	± 0.9659258263 ± 0.7071067812 ± 0.2588190451
7	-0.2225209340 -0.6234898019 -0.9009688679 -1.0000000000	± 0.9749279122 ± 0.7818314825 ± 0.4338837391 0.0000000000
8	-0.1950903220 -0.5555702330 -0.8314696123 -0.9807852804	± 0.9807852804 ± 0.8314696123 ± 0.5555702330 ± 0.1950903220
9	-0.1736481777 -0.5000000000 -0.7660444431 -0.9396926208 -1.0000000000	± 0.9848077530 ± 0.8660254038 ± 0.6427876097 ± 0.3420201433 0.0000000000
10	-0.1564344650 -0.4539904997 -0.7071067812 -0.8910065242 -0.9876883406	± 0.9876883406 ± 0.8910065242 ± 0.7071067812 ± 0.4539904997 ± 0.1564344650

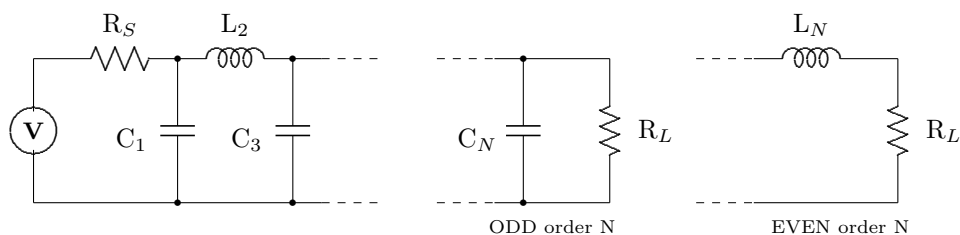
$$P_n(s) = \sum_{j=0}^n c_j s^j \quad (1)$$

$$= c_0 + c_1 s^1 + \dots + c_n s^n \quad (2)$$

Butterworth Polynomial Coefficients

Order	Coefficient	Value
1	c ₀	1.0000000000
	c ₁	1.0000000000
2	c ₀	1.0000000000
	c ₁	1.4142135624
	c ₂	1.0000000000
3	c ₀	1.0000000000
	c ₁	2.0000000000
	c ₂	2.0000000000
	c ₃	1.0000000000
4	c ₀	1.0000000000
	c ₁	2.6131259298
	c ₂	3.4142135624
	c ₃	2.6131259298
	c ₄	1.0000000000
5	c ₀	1.0000000000
	c ₁	3.2360679775
	c ₂	5.2360679775
	c ₃	5.2360679775
	c ₄	3.2360679775
	c ₅	1.0000000000
6	c ₀	1.0000000000
	c ₁	3.8637033052
	c ₂	7.4641016141
	c ₃	9.1416201727
	c ₄	7.4641016141
	c ₅	3.8637033052
	c ₆	1.0000000000
7	c ₀	1.0000000000
	c ₁	4.4939592074
	c ₂	10.097834679
	c ₃	14.591793886
	c ₄	14.591793886
	c ₅	10.097834679
	c ₆	4.4939592074
	c ₇	1.0000000000

Order	Coefficient	Value
8	c ₀	1.0000000000
	c ₁	5.1258308955
	c ₂	13.137071185
	c ₃	21.846150969
	c ₄	25.688355931
	c ₅	21.846150969
	c ₆	13.137071185
	c ₇	5.1258308955
	c ₈	1.0000000000
9	c ₀	1.0000000000
	c ₁	5.7587704831
	c ₂	16.581718739
	c ₃	31.163437478
	c ₄	41.986385733
	c ₅	41.986385733
	c ₆	31.163437478
	c ₇	16.581718739
	c ₈	5.7587704831
	c ₉	1.0000000000
10	c ₀	1.0000000000
	c ₁	6.3924532215
	c ₂	20.431729095
	c ₃	42.802061069
	c ₄	64.882396270
	c ₅	74.233429257
	c ₆	64.882396270
	c ₇	42.802061069
	c ₈	20.431729095
	c ₉	6.3924532215
	c ₁₀	1.0000000000



Normalized Butterworth Filter Component Values

($R_S = R_L = 1\Omega$)

(Capacitance in Farads, Inductance in Henrys)

Order	Component	Value
1	C_1	2.0000000000
2	C_1	1.4142135624
	L_2	1.4142135624
3	C_1	1.0000000000
	L_2	2.0000000000
	C_3	1.0000000000
4	C_1	0.7653668647
	L_2	1.8477590650
	C_3	1.8477590650
	L_4	0.7653668647
5	C_1	0.6180339887
	L_2	1.6180339887
	C_3	2.0000000000
	L_4	1.6180339887
	C_5	0.6180339887
6	C_1	0, 5176380902
	L_2	1.4142135624
	C_3	1.9318516526
	L_4	1.9318516526
	C_5	1.4142135624
	L_6	0.5176380902
7	C_1	0.4450418679
	L_2	1.2469796037
	C_3	1.8019377358
	L_4	2.0000000000
	C_5	1.8019377358
	L_6	1.2469796037
	C_7	0.4450418679

Order	Component	Value
8	C_1	0.3901806440
	L_2	1.1111404660
	C_3	1.6629392246
	L_4	1.9615705608
	C_5	1.9615705608
	L_6	1.6629392246
	C_7	1.1111404660
	L_8	0.3901806440
9	C_1	0.3472963553
	L_2	1.0000000000
	C_3	1.5320888862
	L_4	1.8793852416
	C_5	2.0000000000
	L_6	1.8793852416
10	C_1	0.3128689301
	L_2	0.9079809995
	C_3	1.4142135624
10	L_4	1.7820130484
	C_5	1.9753766812
	L_6	1.9753766812
	C_7	1.7820130484
	L_8	1.4142135624
	C_9	0.9079809995
	L_{10}	0.3128689301