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# Problems and Solutions

*in Mathematics, Physics and Applied Sciences*

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## Resistance of an Infinite Ladder

We want to compute the input resistance to a resistor ladder network of the form in Figure 1. This network can be thought of as an infinite train of 'L' sections, each with a horizontal

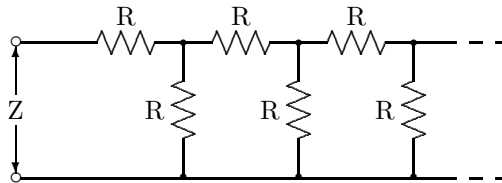


Figure 1: Infinite Resistor Ladder

arm of  $R$  ohms and a vertical arm of  $R$  ohms.

If the input resistance is finite and non-zero, then adding one more section to the front will not change the resistance of the network. Hence, if the resistance is  $Z$  ohms, we find that

$$Z = R + R \parallel Z.$$

In other words, taking the original ladder resistance,  $Z$  in parallel with 'R' and adding another series  $R$  will still give the resistance,  $Z$ . Using the equation for parallel resistances, we find

$$Z = R + \frac{RZ}{R+Z} \tag{1}$$

$$Z^2 + RZ = 2RZ + R^2 \tag{2}$$

$$Z^2 - RZ - R^2 = 0 \tag{3}$$

This quadratic in  $Z$  can be solved for  $Z$ , yielding

$$Z = \frac{(1 + \sqrt{5})}{2} R = 1.618 R$$

Variations of this problem are possible. For example, the ladders in Figures 2 and 3 have input resistances of  $2.732 R$  and  $1.618 R$ , respectively. These values can be computed from the same technique used to get the resistance of Figure 1.

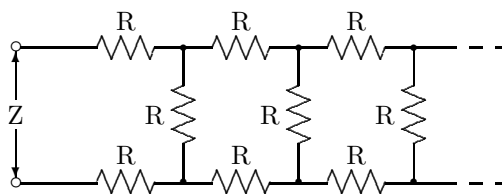


Figure 2: Ladder Network #2

It is curious that the network in Figure 1 has the same input resistance as the network in Figure 3, even though the topologies are different. This result can be explained by observing the symmetry in the latter figure which demands that its input resistance be unchanged if it is turned upside down, *i.e.* if the input terminals are reversed.

Because of this symmetry, we can create an additional section which leaves the original network impedance unchanged by using one parallel resistor and one series resistor and performing the reversal. But this added section (minus the reversal) is the same as that used to duplicate the first network — hence the network equations are the same!

Two networks with the same network equations will clearly have the same properties and are electrically identical, in spite of the topological differences.

Furthermore, the differences in topology are less than imagined at first sight. If, in the first figure, we add a parallel resistance to the input terminals and then add the series resistance to the lower terminal, instead of the upper terminal, the resistance of the network will be unchanged. If we alternately add the series resistance to the upper and lower terminals as more sections are added, the topology of the resulting network is the same as that of Figure 3.

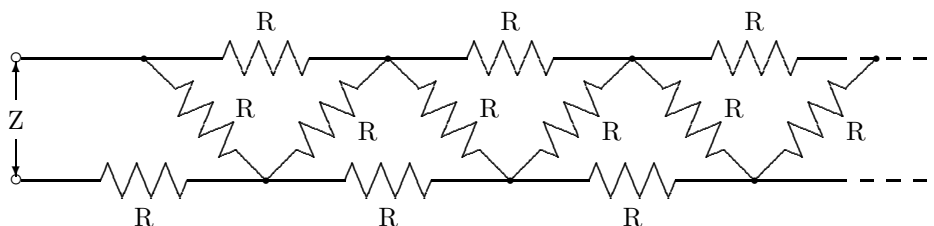


Figure 3: Ladder Network #3