Problems and Solutions

in Mathematics, Physics and Applied Sciences

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Computing the 3 dB Point for an All-Pole Magnitude Response

A common problem in circuit theory is to determine certain reference frequencies at which response functions have significantly changed from their design values. One of the most used reference points is the frequency at which a response function has dropped to 1/2 its maximum or initial value.

The traditionally used reference value, 3dB, is a simple approximation to

$$10\log_{10} 2 = 3.01029995664...$$

or, if the response is *down* 3dB,

$$10\log_{10}\frac{1}{2} = -3.01029995664...$$

Note that this is a power ratio in circuit theory so the 3dB point is that point at which the ratio of the maximum power to this reference power is 2 (or 1/2). For voltages, these ratios are $\sqrt{2}$ or $\sqrt{1/2}$. The appropriate equation is then $20\log_{10}\sqrt{2} = 3.01029995664...$ or $20\log_{10}\sqrt{\frac{1}{2}} = -3.01029995664...$

To determine the frequency at which the magnitude (voltage) of an all-pole filter response has dropped to $\sqrt{1/2} \approx 0.707$ (-3 dB) of its value at DC, simply change the sign of the constant term of the filter polynomial in ω^2 and find the largest real root of the resulting function.

Suppose the magnitude response function looks like this:

$$|H(j\omega)| = \frac{k}{\sqrt{\omega^{2n} \pm a\omega^{2n-2} \pm \cdots \pm p\omega^2 + k^2}}.$$

The polynomial of interest is under the square root sign. If we solve the following modified equation for its maximum real zero

$$\omega^{2n} \pm a\omega^{2n-2} \pm \cdots \pm p\omega^2 - k^2 = 0$$

then the value of the sum of the terms containing ω will be k^2 at that frequency. Hence, substituting in the original equation we have

$$|H(j\omega)|_{3dB} = \frac{k}{\sqrt{\omega_{(3dB)}^{2n} \pm a\omega_{(3dB)}^{2n-2} \pm \cdots \pm p\omega_{(3dB)}^2 + k^2}} \quad (1)$$

$$= \frac{k}{\sqrt{k^2 + k^2}} \quad (2)$$

$$= \frac{1}{\sqrt{2}} \quad (3)$$