Problems and Solutions

in Mathematics, Physics and Applied Sciences

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Design Notes: An Inductive Bridged 'T' Notch Filter

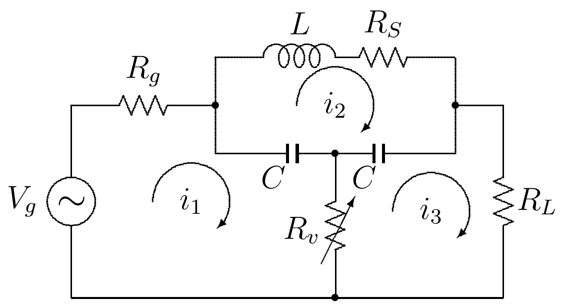

Figure 1: Inductive Bridged 'T' Notch Filter

The above figure is a schematic diagram for simple notch filter often used as an example in circuit synthesis textbooks. The purpose of this treatment is to call attention to some labor saving devices and tips which reduce the design effort in specifying the component values for the required notch frequency.

It is well known that circuits with the above topology can be solved using mesh or nodal techniques and linear algebra. For example, using KVL and the above three current loops, we can completely specify the relationships between all network variables with the matrix representation

$$\mathbf{Z} \cdot \mathbf{I} = \mathbf{V},$$

where $\mathbf{Z}$ is the impedance matrix, $\mathbf{I}$ is the vector of loop currents, and $\mathbf{V}$ is the vector of loop sources.

The first step in the solution process is to write the network impedance matrix:

$$\mathbf{Z} = \begin{vmatrix} R_g + 1/sC + R_v & -1/sC & -R_v \\ -1/sC & sL + R_S + 2/sC & -1/sC \\ -R_v & -1/sC & R_v + 1/sC + R_L \end{vmatrix}$$

The complete network is specified from

$$\begin{vmatrix} R_g + 1/sC + R_v & -1/sC & -R_v \\ -1/sC & sL + R_S + 2/sC & -1/sC \\ -R_v & -1/sC & R_v + 1/sC + R_L \end{vmatrix} \begin{vmatrix} i_1 \\ i_2 \\ i_3 \end{vmatrix} = \begin{vmatrix} V_g \\ 0 \\ 0 \end{vmatrix}$$

We are interested in determining the value of R_v such that a null will occur in the output at a specified frequency f. At the notch frequency, the current in R_L is zero. Solving for that current, i_3,

$$i_3 = \frac{\begin{vmatrix} R_g + 1/sC + R_v & -1/sC & V_g \\ -1/sC & sL + R_S + 2/sC & 0 \\ -R_v & -1/sC & 0 \end{vmatrix}}{\Delta}$$

so that,

$$i_3 = \frac{V}{\Delta}(1/s^2C^2 + sR_vL + R_LR_S + 2R_v/sC),$$

where Δ is the determinant of the impedance matrix, $\mathbf{Z}$. The first key observation is that there is no need to calculate the determinant of the impedance matrix. We are simply looking for the conditions which force the numerator to zero. Hence, after some simplification

$$s^3LC^2R_v + s^2C^2R_SR_v + s2CR_v + 1 = 0.$$

Replacing s with $j\omega$ and collecting real and imaginary terms, we have

$$(1 - \omega^2C^2R_SR_v) + j\omega CR_v(2 - \omega^2LC) = 0$$

The second key observation is that the real and imaginary terms in this equation must *both* be zero if there is to be a transmission null. We can pull the required value of R_v from the derived equations easily. First, the null frequency is found from the imaginary portion to be

$$\omega = \sqrt{\frac{2}{LC}}. \tag{1}$$

Substituting this value of ω in the real part we find R_v

$$R_v = \frac{L}{2R_S C} \tag{2}$$

From the preceding, we find that the inductive bridged 'T' network will exhibit a transmission null (frequency notch) at $f = \omega/2\pi$, and that R_v must be set to the value given in (2).