Problems and Solutions

in Mathematics, Physics and Applied Sciences

compiled by C. Bond — Vol.1 no.23

Design Notes: RC Phase Shift Oscillator

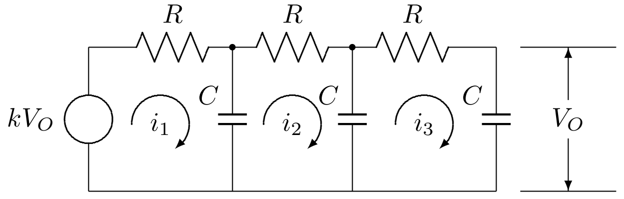

Figure 1: RC Phase Shift Network

The RC network above, when configured as shown, will oscillate at a characteristic frequency if the amplifier gain is sufficient. The amplifier, indicated as a generator on the left of the diagram, is a voltage controlled voltage source (VCVS) driven by the output voltage V_O.

This note suggests a simple method for determining the oscillating frequency, f, and required gain, k, in terms of the circuit elements.

The first step is to write out the network equations in matrix form. This gives:

$$\begin{vmatrix} R+1/sC & -1/sC & 0 \\ -1/sC & R+2/sC & -1/sC \\ 0 & -1/sC & R+2/sC \end{vmatrix} \begin{vmatrix} i_1 \\ i_2 \\ i_3 \end{vmatrix} = \begin{vmatrix} kV_O \\ 0 \\ 0 \end{vmatrix}$$

Symbolically, this is equivalent to

$$\mathbf{Z} \cdot \mathbf{I} = \mathbf{V}$$

The RC network produces a phase lag and, to get the positive feedback required for oscillations, we will use an inverting amplifier and find the frequency corresponding to a 180 degree lag. The attenuation at this frequency sets a lower bound on the gain needed to sustain the oscillations.

Solving for current i_3,

$$i_3 = \frac{\begin{vmatrix} R+1/sC & -1/sC & kV_O \\ -1/sC & R+2/sC & 0 \\ 0 & -1/sC & 0 \end{vmatrix}}{\Delta} \tag{1}$$

$$= \frac{kV_O}{s^2C^2\Delta} \tag{2}$$

where Δ is the determinant of the impedance matrix, $\mathbf{Z}$. But $i_3 = V_O \cdot sC$ so,

$$s^3C^3\Delta = k \tag{3}$$

$$s^3R^3C^3 + 5s^2R^2C^2 + 6sRC + 1 = k \tag{4}$$

Substituting $j\omega$ for s and collecting real and imaginary parts,

$$(1 - 5\omega^2R^2C^2) + j\omega RC(6 - \omega^2R^2C^2) = k. \tag{5}$$

Setting the imaginary part to 0 and solving for ω

$$\omega = \frac{\sqrt{6}}{RC}.$$

Substituting this value in the real part of (5),

$$k = -29. \tag{6}$$

Hence, the circuit will oscillate at frequency $f = \omega/2\pi$. The amplifier must be in an inverting configuration (negative sign) and have a gain of (at least) 29, in order to support oscillations.