Problems and Solutions

in Mathematics, Physics and Applied Sciences

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Design Notes: Twin ‘T’ RC Notch Filter

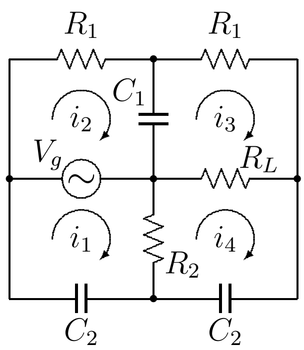

Figure 1: RC Twin ‘T’ Notch Filter

The schematic diagram above will be used for the analysis and design of an RC notch filter. The ‘standard’ version of this filter, as presented in many textbooks and virtually all diagrams on the web, has a specific relationship between the component values which may not be optimum for all applications.[1] In this treatment a somewhat more general approach is taken, and a more flexible relation is derived.

[1]Specifically, $R_2 = R_1/2$ and $C_2 = 2C_1$.

We begin by writing the matrix equations for the network.

$$\begin{vmatrix} R_2+1/(sC_2) & 0 & 0 & -R_2 \\ 0 & R_1+1/(sC_1) & -1/(sC_1) & 0 \\ 0 & -1/(sC_1) & R_1+R_L+1/(sC_1) & -R_L \\ -R_2 & 0 & -R_L & R_2+R_L+1/(sC_2) \end{vmatrix} \begin{vmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \end{vmatrix} = \begin{vmatrix} V_g \\ -V_g \\ 0 \\ 0 \end{vmatrix}$$

Symbolically, this is $\mathbf{Z} \cdot \mathbf{I} = \mathbf{V}$, where $\mathbf{Z}$ is the impedance matrix, $\mathbf{I}$ is the loop current vector and $\mathbf{V}$ is the vector of source generators.

At null, the currents in R_L from loops 3 and 4 will cancel. Using the directions assumed in the schematic, this means that $i_3 = i_4$.

From the network equations,

$$i_3 = \frac{\begin{vmatrix} R_2+1/(sC_2) & 0 & V_g & -R_2 \\ 0 & R_1+1/(sC_1) & -V_g & 0 \\ 0 & -1/(sC_1) & 0 & -R_L \\ -R_2 & 0 & 0 & R_2+R_L+1/(sC_2) \end{vmatrix}}{\Delta}$$

and,

$$i_4 = \frac{\begin{vmatrix} R_2+1/(sC_2) & 0 & 0 & V_g \\ 0 & R_1+1/(sC_1) & -1/(sC_1) & -V_g \\ 0 & -1/(sC_1) & R_1+R_L+1/(sC_1) & 0 \\ -R_2 & 0 & R_L & 0 \end{vmatrix}}{\Delta}$$

The null conditions we are interested in can be found by expanding the numerators of the above expressions for i_3 and i_4 and setting them equal. After considerable algebra, and separation of the real and imaginary parts, this reduces to

$$\left(R_1^2 R_2 - \frac{2R_2}{\omega^2 C_1 C_2}\right) + j\left(\frac{1}{\omega^3 C_1 C_2^2} - \frac{2R_1 R_2}{\omega C_1}\right) = 0. \qquad (1)$$

For the above equation, both the real and imaginary parts must be zero. The notch frequency (transmission null) can be found by setting the imaginary part to zero and solving for ω. We find,

$$\omega^2 = \frac{1}{2R_1 R_2 C_2^2} \qquad (2)$$

Setting the real part to zero and subsituting the value found for ω^2 gives,

$$R_1C_1 = 4R_2C_2. \tag{3}$$

Note that this relation between the components in the two 'T' sections is satisfied by the values in the standard configuration, but supports other choices as well. For the standard model, the frequency of the null is given by

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi RC}, \tag{4}$$

but as long as (3) is satisfied, other possibilities are available.