Problems and Solutions

in Mathematics, Physics and Applied Sciences

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A Problem in Integration, #1

A solid sphere is bored out in such a way that the radial axis of the removed cylinder of material passes through the center. The ring of remaining material stands 6 centimeters high. What is the volume of this ring?

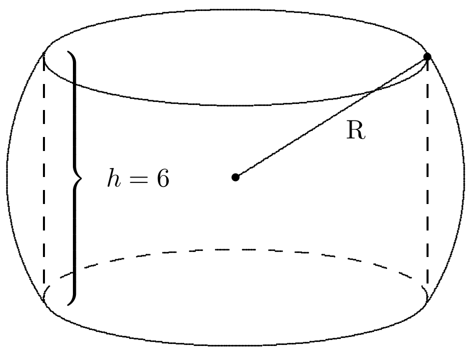

Figure 1: Sphere with Cylindrical Bore

One way to compute the volume of the ring is to subtract the volume of the removed material from the volume of the original sphere. This bored out material can be regarded as a right cylinder with spherical end caps on the two flat surfaces.

Another way is to compute the volume of the partial sphere, excluding the end caps, and then subtract the volume of the right cylinder. We will choose this method.

Referring to Figure 1, the volume of the partial sphere can be computed by using the disk method and integrating from the top edge of the ring to the bottom edge. Using the center of the sphere as the origin, x as the horizontal axis through the center and y the vertical axis, the equation is

$$\begin{aligned} V_s &= \int_{-3}^{+3} \pi\, x^2\, dy && (1) \\ &= \int_{-3}^{+3} \pi (R^2 - y^2)\, dy. && (2) \end{aligned}$$

The right cylinder has volume

$$\begin{aligned} V_c &= 6\pi r^2 && (3) \\ &= 6\pi (R^2 - 9) && (4) \end{aligned}$$

where r is the radius of the cylinder. Note the requirement: $R \geq 3$.

So the volume of the ring is $V = V_s - V_c$. Subtracting (4) from (2)

$$\begin{aligned} V &= \pi \int_{-3}^{+3} (R^2 - y^2)\, dy - 6\pi(R^2 - 9) && (5) \\ &= \pi \left[R^2 y - \frac{y^3}{3} \right]_{-3}^{+3} - 6\pi R^2 + 54\pi && (6) \\ &= \pi(3R^2 - 9 + 3R^2 - 9) - 6\pi R^2 + 54\pi && (7) \\ &= 36\pi && (8) \end{aligned}$$

Surprisingly, this result is independent of the radius of the sphere (within limits). As long as the radius $R \geq 3$ the result holds. Hence, another way to compute the volume of the ring is to compute the volume of a sphere with $R = 3$ representing the case of an infinitesimal bored out volume. This sphere, of course, has volume given by

$$V = \frac{4}{3}\pi R^3 = 36\pi \quad (9)$$