Engineering Notebook

miscellaneous problems and solutions

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Rotary Actuator Equations

Actuator with zero bend angle.

Given two points at which the track radius and gap skew angle (azimuth) are known, it is possible to determine the actuator radius, the distance between the actuator center and the disk spindle center, and the skew angle at any track. In Figure (1), r_i and r_o represent an inner and outer radius with known skew angles.

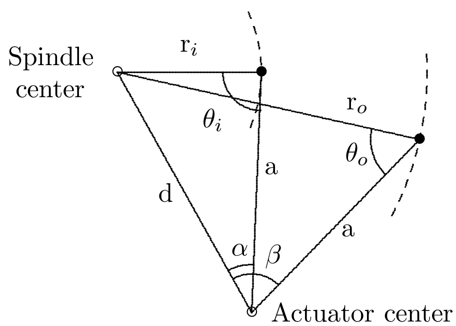

Figure 1: Actuator layout diagram

The cosine law can be used to compute the distance, d, between the actuator and spindle centers. Let ϕ_i and ϕ_o be the measured skew angles at radii r_i and r_o, respectively. We use the convention that $\theta_i = \pi/2 + \phi_i$ and $\theta_o = \pi/2 + \phi_o$. With this convention, skew angles at increasing radii become more negative.

Then,

$$d^2 = r_i^2 + a^2 - 2r_i a\cos(\theta_i) \quad (1)$$

$$d^2 = r_i^2 + a^2 - 2r_i a\cos(\pi/2 + \phi_i) \quad (2)$$
$$d^2 = r_i^2 + a^2 + 2r_i a\sin(\phi_i). \quad (3)$$

Hence,

$$d = \sqrt{r_i^2 + a^2 + 2r_i a\sin(\phi_i)} \quad (4)$$

Using $d^2 = r_o^2 + a^2 + 2r_o a\sin(\phi_o)$ and Equation (3), we can solve for a.

$$a = \frac{r_o^2 - r_i^2}{2(r_i\sin(\phi_i) - r_o\sin(\phi_o))} \quad (5)$$

Since the last equation will be true for any radius/skew angle number pairs, we have the general equation

$$a = \frac{r_o^2 - r_t^2}{2(r_t\sin(\phi_t) - r_o\sin(\phi_o))} \quad (6)$$

To find the skew angle at an arbitrary radius,[1] solve Equation (3) for the angle.

$$\sin(\phi_t) = \frac{d^2 - r_t^2 - a^2}{2r_t a} \quad (7)$$

Hence,

$$\phi_t = \sin^{-1}\left(\frac{d^2 - r_t^2 - a^2}{2r_t a}\right) \quad (8)$$

If we assume that r_i is the innermost radius and r_o is the outermost radius, the total span of the actuator in degrees is given by:

$$\text{span}_{tot} = \beta - \alpha \quad (9)$$

where α and β can be determined by using the sine law.

$$\beta = \sin^{-1}(r_o\sin(\theta_o)/d)$$
$$\alpha = \sin^{-1}(r_i\sin(\theta_i)/d)$$

For the case under consideration in which the bend angle is zero, the radius for which the skew angle is zero is simply,

$$r_z = \sqrt{d^2 - a^2)} \quad (10)$$

[1]Note that the radius is constrained by the following relation: $(d - a) < r_t < (d + a)$.

Actuator with known bend angle.

If the bend angle is known, we can solve for the skew angle at any radius from two points with known radius and skew angle, as previously shown. All that is necessary is to add or subtract the bend angle, B, from the measured skew angles at the appropriate points in the calculations.

Actuator with unknown bend angle.

For the case of an unknown bend angle, we require an additional point at which the radius and skew angle are known. Let the radius be r_m and the skew angle be ϕ_m.

The given (or measured) skew angles, ϕ_i, ϕ_m and ϕ_o, which are referred to the disk track, consist of a component due to the actuator rotary angle and a component due to the bend angle. To separate these components, let

$$\begin{aligned}\phi_i &= s_i + B \\ \phi_m &= s_m + B \\ \phi_o &= s_o + B\end{aligned}$$

where the s_n are the rotary skew angles and B is the bend angle. Equation (6) is valid for any pair of tracks, but only for the case of zero bend angle. Nevertheless, we can eliminate the actuator radius, a, if we have three given points by using the (unknown) rotary skew angles, s_n, as shown.

$$\frac{r_o^2 - r_i^2}{2(r_i \sin(s_i) - r_o \sin(s_o))} = \frac{r_o^2 - r_m^2}{2(r_m \sin(s_m) - r_o \sin(s_o))}. \tag{11}$$

Rearranging,

$$\frac{r_o^2 - r_i^2}{r_o^2 - r_m^2} = K = \frac{r_i \sin(s_i) - r_o \sin(s_o)}{r_m \sin(s_m) - r_o \sin(s_o)} \tag{12}$$

where K is introduced to simplify the rest of the derivation.

From Equation (12), we have

$$K r_m \sin(s_m) - K r_o \sin(s_o) = r_i \sin(s_i) - r_o \sin(s_o). \tag{13}$$

Now replacing the s_n with $\phi_n - B$,

$$K r_m \sin(\phi_m - B) - K r_o \sin(\phi_o - B) = r_i \sin(\phi_i - B) - r_o \sin(\phi_o - B). \tag{14}$$

Expanding by the sum-of-angles formula,

$$Kr_m \sin(\phi_m)\cos(B) - Kr_m \cos(\phi_m)\sin(B) - Kr_o \sin(\phi_o)\cos(B) + Kr_o \cos(\phi_o)\sin(B) = r_i \sin(\phi_i)\cos(B) - r_i \cos(\phi_i)\sin(B) - r_o \sin(\phi_o)\cos(B) + r_o \cos(\phi_o)\sin(B).$$

Dividing by $\cos(B)$,

$$Kr_m \sin(\phi_m) - Kr_m \cos(\phi_m)\tan(B) - Kr_o \sin(\phi_o) + Kr_o \cos(\phi_o)\tan(B) = r_i \sin(\phi_i) - r_i \cos(\phi_i)\tan(B) - r_o \sin(\phi_o) + r_o \cos(\phi_o)\tan(B).$$

Collecting terms,

$$\tan(B) = \frac{(K-1)r_o \sin(\phi_o) - Kr_m \sin(\phi_m) + r_i \sin(\phi_i)}{(K-1)r_o \cos(\phi_o) - Kr_m \cos(\phi_m) + r_i \cos(\phi_i)}. \tag{15}$$

The bend angle, B, can now be computed from the measured number pairs. Furthermore, we can now compute the actual rotary component of skew, which is the measured (given) value minus the bend angle: $\phi_n - B$.

The radius of the actuator can still be computed from Equation (6) provided we use the actual rotary component of skew, s_i and s_o,

$$a = \frac{r_o^2 - r_i^2}{2(r_i \sin(s_i) - r_o \sin(s_o))} \tag{16}$$

To compute the distance between the actuator center and the spindle center we can use Equation (4) with the same angle substitution strategy we used to compute a.

$$d = \sqrt{r_i^2 + a^2 + 2r_i a \sin(s_i)} \tag{17}$$

To find the skew angle at any radius, we must modify Equation (8) so that it gives the rotary component of skew,

$$s_t = \sin^{-1}\left(\frac{d^2 - r_t^2 - a^2}{2r_t a}\right) \tag{18}$$

Then the measured skew will be $s_t + B$.

Determining the radius at which the measured skew angle will be zero is not as simple as in the case for zero bend angle. We can work from Equation (7), which gives a general relation between the skew angle and track radius when d and a are known. Recalling that the equation is valid for the rotary component of skew only, we note that when the measured angle is to be zero, the rotary skew must be $-B$.

Then, using r_z for the radius at zero (measured) skew angle,

$$\sin(-B) \quad = \quad \frac{d^2 - r_z^2 - a^2}{2r_z a}$$

This is a quadratic equation in r_z. Solving for r_z,

$$r_z = a\sin(B) \pm \sqrt{a^2\sin^2(B) + (d^2 - a^2)} \qquad (19)$$

where the positive sign is the correct choice, because the result we are seeking (linear distance) is always non-negative.