Finding the Focal Point of a Spherical Mirror or Lens

by C. Bond

The *focal point* of a concave mirror is that point at which light rays from a distant object are expected to converge. It is a central concept in the characterization of both mirrors and thin lenses.

For our derivation, we observe that it is only necessary to consider a cross-section of the mirror which passes through the center of curvature. By symmetry, the behavior of distant rays which strike the mirror elsewhere will be the same.

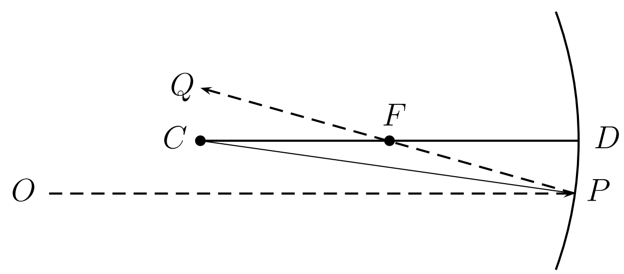

Figure 1: Ray Diagram

In the diagram, C is the center of the sphere which forms the contour of the mirror. $\overline{CD}$ is a radius to the center of the arc. A ray from a distant object, $\overline{OP}$, parallel to $\overline{CD}$ is reflected as ray $\overline{PQ}$ intersecting $\overline{CD}$ at F.

$\overline{CP}$ is a radial line from the center of the sphere and is therefore normal to the surface at P. The angle of incidence is equal to the angle of reflection, so $\angle OPC = \angle CPQ$. Let the angle of incidence $= \alpha$. Then $\angle CPQ = \alpha$ and $\angle PCD = \alpha$. $\angle PFD = 2\alpha$.

Now for the approximations. Assume the ray $\overline{OP}$ is very close to $\overline{CD}$. In this case the arc $\overline{DP}$ is short and very nearly a straight line perpendicular to $\overline{CD}$. With this approximation, $\tan(\angle DFP) = \tan(2\alpha) \approx \dfrac{\overline{DP}}{\overline{FD}}$. Also, $\tan(\angle DCP) = \tan(\alpha) \approx \dfrac{\overline{DP}}{\overline{CD}}$.

With $\overline{OP}$ close to $\overline{CD}$ the angles α and 2α are small. Hence, $\tan(\alpha) \approx \alpha$ and $\tan(2\alpha) \approx 2\alpha$. So $\frac{\overline{DP}}{\overline{FD}} \approx 2\frac{\overline{DP}}{\overline{CD}}$, or $\overline{CD} \approx 2\overline{FD}$. This places point F at the midpoint of radius $\overline{CD}$.

We have now shown that rays from distant objects whose paths are parallel to and sufficiently close to the radius through the center of a spherical mirror will, after reflection, pass through a common point, F, whose distance from the mirror is 1/2 the radius. This point is called the *focal point*.

In the literature, the focal point is usually identified with the symbol f, and the equation

$$f = \frac{r}{2},$$

where r is the radius of curvature is used.