Derivation of Focal Relations

A central equation in optics is the equation relating *focal length*, *object distance* and *image distance*. This equation is usually expressed as

$$\frac{1}{f} = \frac{1}{p} + \frac{1}{q}, \tag{1}$$

where f is the focal length, p is the distance from the object to the lens, and q is the distance from the lens to the image.

We can derive this equation from the following figure using simple geometry.

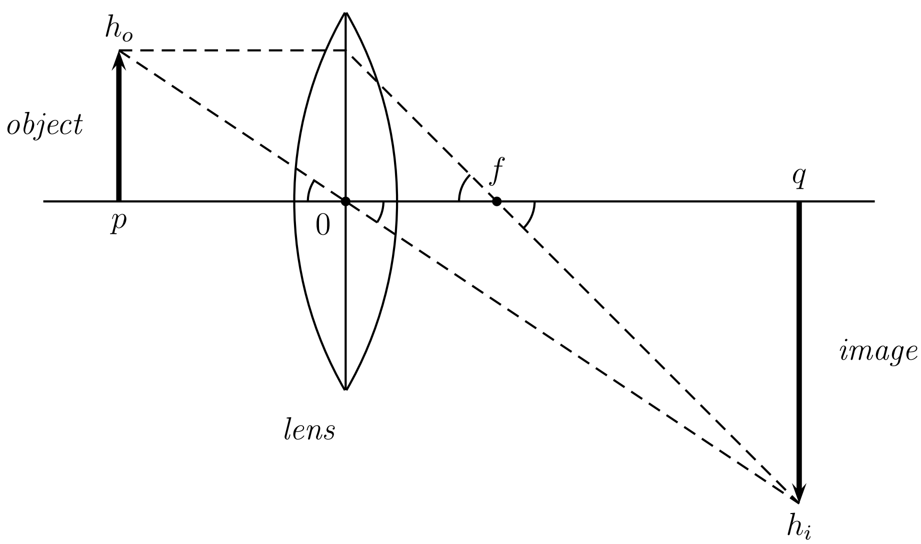

Figure 1: Lens, Object and Image

In the figure, the center of the lens is at 0. p is the distance from the lens to the object and q is the distance from the lens to the image. h_o is the height of the object and h_i is the height of the image. f is the distance from the lens to the focal point.

Using the similar triangles indicated by the alternate interior angles at 0, consisting of sides p and h_o for the left and q and h_i for the right, we find

$$\frac{h_o}{h_i} = \frac{p}{q}. \tag{2}$$

Likewise, from the similar triangles indicated by the alternate interior angles at f, consisting of h_o along the center line of the lens and f for the left and h_i and $q - f$ for the right, we find

$$\frac{h_o}{h_i} = \frac{f}{q-f}. \tag{3}$$

Hence,

$$\begin{aligned}
\frac{p}{q} &= \frac{f}{q-f} && (4)\\
p \cdot q - p \cdot f &= q \cdot f && (5)\\
p \cdot q &= f(p+q) && (6)\\
f &= \frac{p \cdot q}{p+q} && (7)
\end{aligned}$$

Finally,

$$\frac{1}{f} = \frac{p+q}{p \cdot q} = \frac{1}{p} + \frac{1}{q}. \tag{8}$$

.