Snell’s Law
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It is believed that Snell developed his famous equation by purely empirical means. He made numerous measurements of the refracting properties of various materials and found a relationship which made accurate predictions. Later, it was found that his result could be proven.

This proof of Snell’s law is purely geometric and only requires the initial assumption that the index of refraction, $n$, is related to the speed of light in the media by the following relation:

$$n = \frac{c}{v}$$

where $v$ is the speed of light in the medium. The geometry is illustrated in Fig. (1), with two parallel rays approaching the interface at an angle, $\theta_i$, from the normal. Media $m_1$ and $m_2$ have indices of refraction, $n_i$ and $n_r$, respectively.

Snell’s law is usually stated as,

$$n_i \sin \theta_i = n_r \sin \theta_r,$$

where $i$ is an incident ray and $r$ is a refracted ray.

![Figure 1: Plane Wave Incident on Interface](image-url)
If the two rays shown travel together, then \( i_2 \) just reaches point ‘A’ when \( i_1 \) strikes the interface. \( i_2 \) completes the remaining journey to the interface at velocity \( c/n_i \) and covers distance \( x \sin \theta_i \).

Fig. (2) shows the incident and refracted rays with critical points labelled. While \( i_2 \) completes its journey to the interface, \( i_1 \) is refracted into \( m2 \) travelling at the new velocity \( c/n_r \). It reaches point ‘B’ when \( i_2 \) reaches the interface. Note that the acute angle formed by the entry point of \( i_1 \) and the right triangle at ‘A’ is \( \theta_i \), and the corresponding acute angle formed by the entry point of \( r_2 \) and the right triangle at ‘B’ is \( \theta_r \).

Point ‘B’ is found by the intersection of two circles. One is the circle centered at the entry point of \( i_1 \) at the interface and with radius equal to the distance travelled in medium \( m2 \) while \( i_2 \) travels its excess distance to the interface in \( m1 \). The second circle is centered halfway between the entry point of \( i_1 \) and \( i_2 \) along the interface, and with radius equal to half that distance \( x/2 \). This is the locus of right triangles with \( x \) as a hypotenuse.

The ratio of the two distances is the same as the ratio of the sines of angles \( \theta_i/\theta_r \). But this ratio is also the ratio of the velocities of light in the respective media and is therefore inversely proportional to the indices of refraction.
Therefore,

\[ n_i \sin \theta_i = n_r \sin \theta_r \]

as was to be proved.

There are other ways to prove Snell’s law, but the visual appeal of a geometric proof is that the involved quantities and their relationships can be easily seen in the figure.

Snell’s law has counterparts in other physical contexts, where electromagnetic fields exhibit similar properties.