Snell's Law

by C. Bond

It is believed that Snell developed his famous equation by purely empirical means. He made numerous measurements of the refracting properties of various materials and found a relationship which made accurate predictions. Later, is was found that his result could be proven.

This proof of Snell's law is purely geometric and only requires the initial assumption that the *index of refraction*, n, is related to the speed of light in the media by the following relation:

$$n = \frac{c}{v}$$

where v is the speed of light in the medium. The geometry is illustrated in Fig. (1), with two parallel rays approaching the interface at an angle, θ_i, from the normal. Media $m1$ and $m2$ have indices of refraction, n_i and n_r, respectively.

Snell's law is usually stated as,

$$n_i \sin\theta_i = n_r \sin\theta_r,$$

where i is an incident ray and r is a refracted ray.

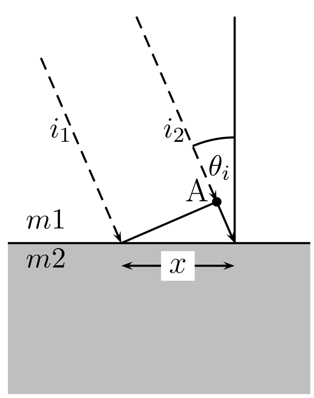

Figure 1: Plane Wave Incident on Interface

If the two rays shown travel together, then i_2 just reaches point 'A' when i_1 strikes the interface. i_2 completes the remaining journey to the interface at velocity c/n_i and covers distance $x \sin \theta_i$.

Fig. (2) shows the incident and refracted rays with critical points labelled. While i_2 completes its journey to the interface, i_1 is refracted into $m2$ trav-

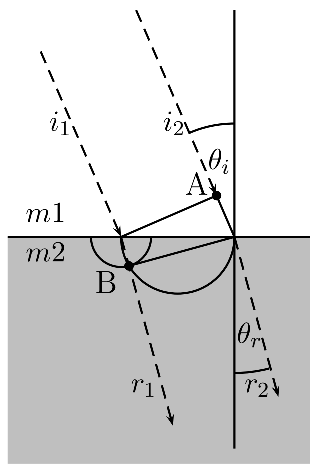

Figure 2: Refracted Wave in Medium

elling at the new velocity c/n_r. It reaches point 'B' when i_2 reaches the interface. Note that the acute angle formed by the entry point of i_1 and the right triangle at 'A' is θ_i, and the corresponding acute angle formed by the entry point of r_2 and the right triangle at 'B' is θ_r.

Point 'B' is found by the intersection of two circles. One is the circle centered at the entry point of i_1 at the interface and with radius equal to the distance travelled in medium m_2 while i_2 travels its excess distance to the interface in m_1. The second circle is centered halfway between the entry point of i_1 and i_2 along the interface, and with radius equal to half that distance $x/2$. This is the locus of right triangles with x as a hypotenuse.

The ratio of the two distances is the same as the ratio of the sines of angles θ_i/θ_r. But this ratio is also the ratio of the velocities of light in the respective media and is therefore inversely proportional to the indices of refraction.

Therefore,

$$n_i \sin\theta_i = n_r \sin\theta_r$$

as was to be proved.

There are other ways to prove Snell's law, but the visual appeal of a geometric proof is that the involved quantities and their relationships can be easily seen in the figure.

Snell's law has counterparts in other physical contexts, where electromagnetic fields exhibit similar properties.